International
Mathematical
Union

IMU Fields Medal 2022
June Huh Interview

Conducted by Andrei Okounkov & Andrei Konyaev

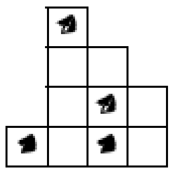
When did you realize that you want to be mathematician? Who and what inspired you to become a mathematician? Was it an easy or difficult decision for you?

Soon after finishing college, I had the good fortune to meet Professor Heisuke Hironaka. It was the first time that I saw someone "doing" mathematics in real time. Before that, mathematics was something inside a book, written by someone from a distant past, at a place that I'd never been. It was like listening to a piece of music for the first time, after pointlessly reading individual notes in its score many times. I often visited his office, and he would tell me long stories on many different things. Unfortunately, I knew too little to digest them, and I could not understand most of what he was saying. Nevertheless, I could follow many of his concrete computations, and I acquired some confidence in dealing with polynomials, power series, and polytopes. After that year, learning and doing mathematics felt very natural to me, and there was no real decision to make.

What kind of mathematics did you like the best in school and at the university? Do you have a your favorite math problem from back then?

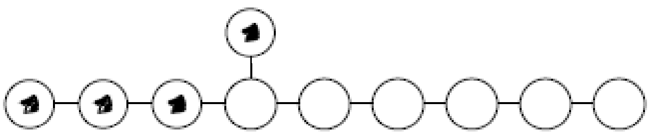
I may be atypical among professional mathematicians in that I was neither very good nor intensely interested in mathematics in school and at the university. I do, however, have a few early experiences that may now be characterized as mathematical. For example, I was playing *The 11th Hour* in middle school, a 90s video game in a horror setting, and there was the following chess puzzle:

Exchange the positions of black and white knights:



After hundreds of ad hoc attempts, spending all my energy for more than a week on this puzzle, I almost gave up. Then I realized that the L-shaped move of the knights and the physical appearance of the irregular chessboard are irrelevant; only the relations between the squares matter. Thus, the problem was equivalent to the problem of exchanging the positions of black and white knights in the following graph, where the knights now move from a vertex to one of its neighbors:

Exchange the positions of black and white knights:



Viewing the same puzzle in this new way, which better reveals the essence of the problem, suddenly the solution was obvious. The two formulations are logically indistinguishable, but our intuition works in only one of them. This made me think about what it means to understand something.

How did you find "your" area of mathematics? What makes it particularly attractive for you?

Loosely speaking, I build spaces from combinatorial objects. Once you have a space to move around, you can use your geometric intuition to extract information hidden in the original combinatorial structure. The discrete (combinatorics) and the continuous (geometry) are two primary modes of human mathematical thinking. I and many other mathematicians find it very satisfying to blur the boundary between them. Why do we feel the satisfaction, I don't know for sure, but we may be all secretly thinking that the distinction is artificial.

How do you choose the problems to work on?

It doesn't feel like I am choosing problems to work on. The problems come to me, in a sense, although others may frame the same process differently. Sometimes I can see them approaching slowly and gradually from distance, and at no specific point do I feel like I'm making conscious choices. In general, it appears to me that I have very little control over what I think. If I sit and do nothing as a mini-experiment, I almost always end up thinking about something that I didn't choose to think about, and it is easy to notice this fact. Essentially the same thing seems to happen on a macroscopic time scale as well, and I find this especially interesting in mathematical contexts. For me, problem-finding and problem-solving are random processes, and there is not much I can or should do, although exposing myself to good books and good people seems to be a good idea. Other than that, the only thing I try is to make myself available as much as possible, as math problems need big open spaces to unfold.

Do you feel that the progress in mathematics is sometimes very fast and sometimes very slow? What do you when it goes fast? What do you when it is slow?

As an individual, the answer is of course "yes". When it's fast, I try to record as much as possible, because my future self may be unable to recover what I could get in such times. When it's slow, there is not much I can do about it other than to wait. And I'm pretty good at waiting. As a species, we seem to be doing well. The progress in mathematics always feels fast, especially while browsing through the daily arXiv postings. We are living in the golden age of mathematics.

What was the first real mathematical result you obtained? Tell us about the result itself, the circumstances, and what effect it had on you.

In 1912, attempting to solve the four-color problem, George Birkhoff associated a polynomial χ_G to a graph G that coherently encodes the answers to the analogous q-color problem for all natural numbers q:

$$\chi_G(q) = (\text{number of ways to color vertices of } G \text{ using } q \text{ colors}).$$

This polynomial, called the *chromatic polynomial*, is a fundamental invariant of graphs. Up to the removal of loops and identification of parallel edges, any other numerical invariant which can be recursively computed by deletion and contraction of edges is a specialization of the chromatic polynomial. In my first paper, I proved that the coefficients of the chromatic polynomial form a log-concave sequence for any graph, resolving a conjecture of Ronald Read from 1968. An important step in the proof is to construct a complex algebraic variety from the given graph and ask a more general question on its topology and singularities.

In general, to any isolated complex hypersurface singularity, the classical works of John Milnor and Bernard Teissier from the 60s and the 70s associate a sequence of numbers that encode the Euler characteristics of all of its general hyperplane sections. Their theory shows that these *Milnor numbers* can be computed by counting the number of solutions to a system of equations determined by the initial singularity. I learned these from Hironaka before moving to the US for graduate school, and I noticed that an analogous theory can be constructed for complex projective hypersurfaces. Teissier has shown that the Milnor numbers of any isolated hypersurface singularity form a *log-convex* sequence, so I tried to see whether something similar can be said about the Milnor numbers of projective hypersurfaces. Interestingly, a careful modification of Teissier's reasoning revealed that the Milnor numbers of any projective hypersurface form a *log-concave* sequence, an opposite conclusion. That was interesting, so I kept that in mind. After I moved to the US, I met Hal Schenck, a combinatorial algebraic geometer who was then working at the University of Illinois at Urbana-Champaign. I took a reading course from him, and I learned about interesting projective hypersurfaces associated with graphs: For a graph G, the hypersurface is defined by

$$\prod_{i\sim j}(x_i - x_j) = 0 \text{ where the product is over all edges of } G.$$

At that point, it was very natural for me to wonder about the graphic interpretation of the Milnor numbers of those projective hypersurfaces. It turned out that the Milnor numbers were exactly the absolute values of the coefficients of the chromatic polynomial of the graph, which was deeply satisfying. Later I learned that the resulting log-concavity was a well-known conjecture in graph theory, so I wrote a paper explaining the above story. In a sense, I knew the solution before I knew the problem. I was lucky.

Can you tell us about your biggest "Eureka!" moment?

I find it interesting that I can remember no such dramatic moments. I now understand a few technical points that I didn't three years ago, say, but it's hard for me to locate the Eureka moments within those three years. The moments I remember are the ones that I realized, probably not for the first time, that I needed to understand those points and, happily, I already did understand those points. Mysteriously, our mind seems to be capable of grasping things that are just beyond our conscious self, and this happens in a place unknown to us. I find the whole process fascinating. Practicing mathematics is the best way to experience the mystery because the difference between knowing and not knowing is most clear in mathematics.

What did you feel and what did you do when you learned about being awarded the prize?

I felt gratitude to my teachers and collaborators. They are the source of all my mathematical outputs. I mostly acted as a vessel of ideas they planted on me.

Who are the people who contributed the most to this success?

My teachers and friends in mathematics. It is difficult to list them because there are many. While browsing through arXiv postings, rambling between shelves of libraries, or sitting in the dining room of an institute, I met heroes and heroines. The last decade of my life was like living a chapter from ancient mythology. There were hundreds of characters, each one of them with a unique set of abilities. It was, and still is, a privilege to have connected with their minds. On clear days, I can see that I am a small and simple part of a big and complex ancient structure, like a mushroom sprouted from a giant underground fungal network. It feels good.

What are the new horizons, new problems, new goals for you now?

My goal is to create and discover beautiful structures; to remain creative and amazed; to understand and to be understood by fellow mathematicians and the greater community.

Outside of mathematics, what are your favorite things to do, interests, pursuits? Do you approach them as a mathematician, or are you happy to forget about mathematics while you are on a break?

I enjoy spending time with my family. When you work on mathematics, you think a lot. You think about schemes, sheaves, and cohomologies. You tend to think about how smart or not smart you are, at least when you are young. You think about that stupid sentence you wrote in your just-submitted paper and worry what the referee will think. Then suddenly you are asked to change diapers, do the dishes, read chapter books, and so on. It really wakes you up.

Of course, in a moment of a great celebration, no one wants to think about the many difficulties human society is facing now. Still, it must be adding some bitterness to your sweet victory. Any words of wisdom on this?

We are well aware of the fact that we are capable of creating great harm and at the same time great good. As our minds become more connected and more aware, I’m hopeful that our better side will eventually dominate, perhaps after many hundreds of years. What is heartbreaking is that many of us are experiencing serious difficulties right now. It matters that each of us tries to steer the flow in the right direction, even when our efforts appear indirect and inconsequential.